CHAPTER 10. CURRENCY SWAPS

“The advent of swaps, as much as anything else, helped transform the world’s segmented capital markets into a single, truly integrated, international capital market.” John F. Marshall and Kenneth R. Kapner (1993)

A currency swap is a contract to exchange two streams of future cash flows in different currencies. Currency swaps were designed in the 1980s to circumvent capital controls imposed by governments and to make borrowing more efficient in global markets. Currency swaps are used to convert debt denominated in one currency into synthetic debt denominated in another currency. Synthetic debt created in this way sometimes allows a segment of the capital market to be tapped that would otherwise not be accessible with debt actually denominated in that currency.

When synthetic debt in a currency is created, we treat it the same as actual debt in that currency when performing analyses of hedging FX exposure. For example, when Fannie Mae issues yen debt but converts it into synthetic US dollar debt by a currency swap, we think of Fannie Mae as having US dollar debt for analytical purposes, even though Fannie Mae nominally has yen debt.

Currency swap positions are also useful in managing FX operating exposure in situations where foreign currency debt or financial assets do not work. A firm with an FX operating exposure (positive or negative) that is too high to be managed solely by foreign currency-denominated debt or financial assets, might use currency swaps in its FX financial hedging strategies.

World Bank–IBM Swap

The first currency swap seems to be a 1981 transaction between the World Bank and IBM. Its details are instructive.

The World Bank wanted to raise additional capital and to denominate the liabilities in Swiss francs because of a low interest rate in that currency. The US market, though, was more receptive to World Bank bonds than the Swiss market, since the World Bank had already saturated the Swiss market for its bonds, and US investors regarded World Bank bonds to have much less credit risk than Swiss investors did. But the US investors wanted to invest in bonds denominated in US dollars.

IBM had financed before by issuing Swiss franc debt, but had since developed the view that the Swiss franc was going to appreciate relative to the US dollar. It wanted to replace its Swiss franc debt with US dollar debt. But of course IBM would incur the transactions costs of issuing new US dollar debt and retiring the Swiss franc debt.

A major global bank observed that both parties could benefit if they made a private deal, termed a currency swap. The swap let IBM receive cash flows of Swiss francs from the World Bank, and the World Bank to receive US dollars from IBM. The World Bank could then go ahead and borrow in US dollars from US investors in the favorable US market, planning to use its US dollar receipts from the currency swap with IBM to make the US dollar bond payments. This way, the Swiss franc swap payments to IBM represented the net effective liability for the World Bank. Similarly, IBM could use the Swiss francs received from the currency swap with the World Bank to meet its Swiss franc debt obligations, while its US dollar payments to the World Bank represented its new effective liability in US...
dollars. Figure 10.1 shows the basic structure of the World Bank-IBM currency swap.

IBM’s motivation was clear. IBM had issued Swiss franc bonds but subsequently wanted to change that liability into a US dollar liability, as it feared an appreciation of the Swiss franc. IBM used the currency swap as an expeditious way to convert Swiss franc debt into US dollar debt, without having to retire its Swiss franc debt and reissue new US dollar debt (avoiding transaction costs). This was money-saving and time-saving.

The World Bank used the currency swap as a capital raising strategy. You might ask why the World Bank didn’t simply issue Swiss franc bonds in the first place, if it wanted its liabilities to be denominated in Swiss francs? The answer is that the World Bank was able to get a lower effective Swiss franc interest rate than it could by issuing Swiss franc bonds directly. There was more appetite for World Bank bonds among US investors than among Swiss investors, and US investors thought the World Bank to be a better credit risk than Swiss investors thought. But the US investors wanted bonds denominated in US dollars, so the World Bank took advantage of the financing opportunity in the US market. It achieved its preferred liability denomination of Swiss francs through the swap deal.

We describe the positions in currency swaps like positions in forward FX contracts. IBM had a long Swiss franc position and a short US dollar position in the swap since it received Swiss francs and paid US dollars. The World Bank had a short Swiss franc position and a long US dollar position in the swap since it paid Swiss francs and received US dollars.
Fixed-for-Fixed Currency Swaps

The basic ("plain vanilla") currency swap is a **fixed-for-fixed swap**. In this case, the cash flows are based upon straight bonds in two currencies, where a straight bond (also called a **bullet bond**) has no features other than promised coupon interest and principal repayment.

The swap stipulates the **time until maturity**, the **two coupon rates**, and the **notional principal** of the swap.

**Example**: two five-year straight bonds, one denominated in US dollars and the other in Swiss francs, both make annual coupon interest payments.  
The coupon rate of the five-year US dollar bond is 6%, and the coupon rate of the five-year Swiss franc bond is 4%.  
With a principal of $1000 for the US dollar bond, the coupon interest payments are 0.06($1000) = $60 per year.  
At maturity, at the same time as the final interest payment, the $1000 in principal payment must also be repaid.

Now consider the equivalent amount of principal in Swiss francs.  
Given an assumed time 0 spot FX rate of 1.60 Sf/$, $1000(1.60 Sf/$) = Sf 1600.  
The Swiss franc bond makes coupon interest payments of 0.04(Sf 1600) = Sf 64 per year, then repays the principal of Sf 1600 at the same time as the last coupon payment.

1. You enter into a three-year **Fixed-For-Fixed Currency Swap**, so that:  
The cash flow stream you are receiving is in Japanese yen and the cash flow stream you are paying is in US dollars.  
The swap has a **notional principal of $1 million**.  
What are the cash flows upon which the currency swap is based, if:  
- The swap is an at-market swap and  
- The three-year par **coupon rates are 5% for US dollars**  
- The three-year par **coupon rates are 2% for yen**  
- The spot FX rate is currently 112.50 ¥/$?

**notional principal in yen** = (112.50 ¥/$)($1 million) = ¥112,500,000

Interest rate of market yield to find yearly coupon payment: (¥112,500,000)(0.02) = ¥2.25 mil

Interest rate of market yield to find yearly coupon payment: ($1,000,000)(0.05) = $50,000

You would be receiving ¥2.25 million per year for three years and a principal payment of ¥112.50 million at year 3. You would pay $50,000 per year for three years and then a payment of $1 million.

Any two counterparties can agree to exchange the cash flows based “notionally” on these two bonds, whether the counterparties actually own the bonds or not.

Counterparty U would agree to receive (from counterparty S) the Swiss franc cash flows of...
Sfr 64 annually for five years, plus Sfr 1600 at maturity. Counterparty S would agree to receive (from counterparty U) the US dollar cash flows of $60 annually for five years, plus $1000 at maturity.

If the coupon rates on the underlying bonds are the same as the market yields for the two bonds, the swap is an **at-market swap**. That is, if 6% is the current market yield on five-year US dollar bonds and 4% is the current market yield on five-year Swiss franc bonds, then the principal of each bond is equal to the present value of its future cash flows. Thus the present values of the swapped cash flows are equivalent at time 0. The two parties are simply exchanging future cash flows worth Sfr 1600 today for future cash flows worth $1000 today, which is fair at the assumed current spot FX rate of 1.60 Sfr/$. In an at-market swap, no time-0 payment is necessary, because the present values of the underlying notional bonds are equivalent, given the current spot FX rate.

**Many currency swaps originate as at-market swaps.**

You enter into a five-year fixed-for-fixed currency swap:
- receive a cash flow stream in British pounds and
- pay a cash flow stream in US dollars.

The swap is an **at-market swap** based on a **notional principal** of $1 million.

What are the cash flows of the swap if:

a) the five-year market yields are 5.50% for US dollars and 9.00% for British pounds,

The cash flow stream you would pay consists of interest payments of 0.055($1 million) = $55,000 per year for five years, plus a final principal payment of $1 million. The cash flow stream you would receive is based on a sterling-denominated bond with a principal amount equal to the pound equivalent of $1 million, given the spot FX rate of 1.50 $/£.

b) and if the spot FX rate is currently 1.50 $/£?

Given the spot FX rate of 1.50 $/£ the principal on the sterling bond is:

\[
\frac{\$1\text{ million}}{1.50 \$/\text{£}} = \text{£666,667}
\]

At a coupon interest rate of 9%, the sterling cash flow receipts consist of interest components of:

\[0.09(\text{£666,667}) = \text{£60,000}\] per year for five years and a principal component of £666,667 at year 5.
**REVIEW OF BOND VALUATION**

The value of a bond is the present value of the future interest and principal payments. The definition of yield-to-maturity is the (annualized) interest rate that discounts the promised future bond payments back to the bond’s present value. Let us find the value of a straight bond (or bullet bond, i.e., with no features other than promised coupon interest and principal repayment) denominated in Swiss francs. Assume the bond has a face value of Sf 100, 4 years to maturity, a 5% annual coupon interest rate, and the yield to maturity is 7%. The cash flows for the bond are annual interest payments of Sf 5 at the end of each of the next 4 years plus a principal repayment of Sf 100 at the end of the 4th year. The present value of those cash flows, given a yield to maturity of 7%, is Sf 5/1.07 + Sf 5/1.07^2 + Sf 5/1.07^3 + Sf 105/1.07^4 = Sf 93.23. For simplicity, the valuation here is for the instant in time after the most recent coupon interest payment. The value might thus be called the “ex-interest” value of the bond. If the face value had been Sf 1000, the present value of the bond would be Sf 932.30, and if the face value had been Sf 10,000, the present value would be Sf 9323. Rather than specify a principal amount or face value, one can simply quote, as the bond market does, on the basis of percentage of face value. Thus, the value of the 4-year, 5% coupon, Swiss franc bond with a 7% yield to maturity is simply quoted as 93.23, with the understanding that this value is a percentage of its face value, which is in Swiss francs. A par bond’s present value is equal to its face value. A bond is thus a par bond if the yield to maturity is equal to the coupon rate. Many coupon bonds are par bonds, or close to it, when they are issued. If the yield to maturity is greater than the coupon rate, the market value of the bond is less than the face value; in this case, the bond is called a discount bond. If the yield to maturity is less than the coupon rate, the value of the bond is greater than the face value; in this case, the bond is called a premium bond.

Example swap rates for at-market swaps, taken from the *Financial Times*, are shown in Table 10.1. These are midpoints of the “bid” and the “ask”.

**TABLE 10.1: CURRENCY SWAP RATES**

<table>
<thead>
<tr>
<th></th>
<th>USD</th>
<th>EUR</th>
<th>JPY</th>
<th>CHF</th>
</tr>
</thead>
<tbody>
<tr>
<td>2Y</td>
<td>5.45</td>
<td>3.10</td>
<td>0.38</td>
<td>1.72</td>
</tr>
<tr>
<td>3Y</td>
<td>5.57</td>
<td>3.26</td>
<td>0.58</td>
<td>1.97</td>
</tr>
<tr>
<td>4Y</td>
<td>5.67</td>
<td>3.46</td>
<td>0.84</td>
<td>2.19</td>
</tr>
<tr>
<td>5Y</td>
<td>5.75</td>
<td>3.65</td>
<td>1.11</td>
<td>2.39</td>
</tr>
<tr>
<td>7Y</td>
<td>5.88</td>
<td>4.03</td>
<td>1.57</td>
<td>2.77</td>
</tr>
<tr>
<td>10Y</td>
<td>6.03</td>
<td>4.44</td>
<td>2.06</td>
<td>3.26</td>
</tr>
</tbody>
</table>

Source: The *Financial Times*

Let’s assume the swap rates in Table 10.1 are current and that you want an at-market five-year currency swap to **Pay Yen** and **Receive US Dollars**. You would pay yen at the coupon rate of 1.41% on the swap’s notional yen principal, and
Receive US dollars at the rate of 6.87% on the equivalent notional principal in US dollars,
At the current spot FX rate.

**Parallel Loans and Back-to-Back Loans**

Currency swaps evolved from **parallel loans**, devised years ago to get around cross-border capital controls. It was illegal by British law for a British company to use capital from the UK for overseas investment. The law was intended to prompt the use of British capital for domestic investment and thus to help create jobs at home. There was no law, however, to prevent a British company from lending British pounds to the local UK subsidiary of a US parent firm. And the US parent firm could lend the equivalent amount in US dollars to the US subsidiary of the British company.

**Parallel Loans:**

- The UK subsidiary of the US firm made pound-denominated interest and principal payments to the British parent company,
- The US subsidiary of the British company would make US dollar interest and principal payments to the US parent firm,
- The US subsidiary of the British company would receive the financing it needed, circumventing Britain’s capital export controls.
- The US firm could effectively repatriate earnings of its UK subsidiary to the United States without paying repatriation taxes to the host government.

Parallel loans thus not only got around capital controls, but were also a way to avoid foreign tax on the repatriated returns of overseas investments.

It isn’t much of a leap to see that even if companies in different countries do not need new capital, they can still use subsidiaries to arrange hypothetical loans to each other and thus accomplish cash flow repatriation.

**Back-to-Back Loan**

- The US subsidiary of a Japanese parent is generating US dollars, and
- The Japanese subsidiary of a US parent is generating yen.
- The Japanese parent’s US subsidiary books a US dollar loan on its balance sheet, payable to the US parent, and then in the future makes US dollar interest and principal payments to the US parent.
- At the same time, the US parent’s Japanese subsidiary books a yen loan on its balance sheet, payable to the Japanese parent, and then makes future yen interest and principal payments to the Japanese parent.

No principal amounts are actually exchanged. This type of an arrangement is called a back-to-back loan.
Exhibit 10.1: balance sheets representing both parallel loan and back-to-back loan arrangements.

The spot FX rate is assumed to be 2 $/£.
The US subsidiary of the UK parent owes $3000 to the US parent and
The UK subsidiary of the US parent owes the equivalent amount in pounds, £1500, to the UK parent.

<table>
<thead>
<tr>
<th>EXHIBIT 10.1: PARALLEL LOANS OR BACK-TO-BACK LOANS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Balance Sheets for Parents and Subsidiaries</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>UK Subsidiary of US Parent</th>
<th>US Parent</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ASSETS</strong></td>
<td><strong>DEBT &amp; EQUITY</strong></td>
</tr>
<tr>
<td>£2500</td>
<td>£1500 Debt to UK Parent</td>
</tr>
<tr>
<td>Subsidiary</td>
<td>£1000 Equity of US Parent</td>
</tr>
<tr>
<td>£2500</td>
<td>£2500</td>
</tr>
<tr>
<td>£1000</td>
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<table>
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<tr>
<th>US Subsidiary of UK Parent</th>
<th>UK Parent</th>
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<tbody>
<tr>
<td><strong>ASSETS</strong></td>
<td><strong>DEBT &amp; EQUITY</strong></td>
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<tr>
<td>$4000</td>
<td>$3000 Debt to US Parent</td>
</tr>
<tr>
<td>Subsidiary</td>
<td>£1000 Equity of UK Parent</td>
</tr>
<tr>
<td>$4000</td>
<td>$4000</td>
</tr>
<tr>
<td>£4000</td>
<td></td>
</tr>
</tbody>
</table>

While parallel loans and back-to-back loans are ways to avoid cross-border capital controls and repatriation taxes, they have some drawbacks.

First, parallel and back-to-back loans are shown on reported balance sheets resulting in higher debt ratios (a key factor in credit ratings).
Second, different legal provisions in different countries make it very difficult to link the loans legally. If one party defaults on its loan, the laws in the other country would still require the other party to pay off the loan on its side of the agreement.
Third, each side had to make full payment of the cash flow; that is, there is no difference check settlement.
Global financial intermediaries serve as brokers so that the various companies no longer have to search for suitable partners. The global banks were able to take the key step to structure the deals as legal swaps of cash flow streams instead of mutual loans, solving the three drawbacks:

1. Swap positions were off balance sheet, and not figured into debt ratio computations.
2. The cash flows could legally be viewed as offsetting legs of a single transaction, which solved the legal problems. In the process, global banks began to serve as swaps dealers, meaning that each counterparty viewed its deal as only with the bank. A bankruptcy by one company would be the bank’s problem, not the other company’s problem.
3. Structuring currency swaps as instruments whose periodic exchanges can be settled with one-way difference checks, like forward FX contracts, relieved the counterparties from exchange of the full amounts of funds at each exchange time. This feature in turn reduced the counterparty risk of default.

**Swap Driven Financing**

The difference in credit risk perceptions of US and European investors was a prime driver of the IBM-World Bank currency swap. Europeans have often seen a strong international company like IBM as having lower credit risk than a supranational agency like the World Bank. US investors typically have had the opposite perception. US investors might require a company like General Electric to pay a higher US dollar interest rate on bonds than the World Bank, while Swiss investors might require the World Bank to pay a higher Swiss franc interest rate on bonds than General Electric. Then, if the World Bank prefers to have a Swiss franc liability, while General Electric prefers to have a US dollar liability, a currency swap can help each organization overcome these asymmetric market perceptions.

**Example:** GE would have to pay an 8% interest rate on US dollar bonds, The World Bank would have to pay only 7% (perceived better credit rating among US investors)
The World Bank would have to pay a 6% interest rate on Swiss franc par bonds  
GE would have to pay only 5% (perceived better credit rating among Swiss investors).  
Their preferred liability denominations without a currency swap:

- GE would pay 8% on US dollar bonds, and the  
- World Bank would pay 6% on Swiss franc bonds.

GE issues Swiss franc bonds at 5%  
World Bank issues US dollar bonds at 7%  
The two organizations engage in a currency swap of 7% US dollars for 5% Swiss francs,  
Each ends up with lower effective financing costs in their preferred currencies.

The 5% incoming Swiss francs from the swap would cover GE’s actual 5% Swiss franc liability,  
GE would effectively be paying US dollar interest at the rate of 7%  
(lower than the 8% it would have to pay on actual US dollar bonds)
World Bank’s 7% US dollar liability would be covered by the currency swap receipts
The net effective liability would be 5% Swiss francs,
(a lower rate than the 6% it would have to pay on actual Swiss franc bonds).

Currency swaps often involve no initial exchange of principal amounts, however, **an exchange may occur when two bond issuers enter a swap**.

When an organization **issues securities to raise capital** and **simultaneously originates a swap as an integral part of the deal**, we call this **swap-driven financing**.

**GE Example**: GE issues synthetic US dollar bonds through
  1. an actual issue of Swiss franc bonds and
  2. a long Swiss franc position in a currency swap.

A company can **create** what we call **synthetic base currency debt** as follows:

**Synthetic Base Currency Debt**
1. Company has actual foreign currency debt.
2. Company takes long swap position in foreign currency.

A **short swap position in the foreign currency** combined with **US dollar debt** effectively creates **synthetic foreign currency debt**:

**Synthetic Foreign Currency Debt**
1. Company has actual base currency debt.
2. Company takes short swap position in foreign currency.

Companies that really want to have foreign currency debt may **sometimes find it more advantageous to issue base currency debt, and then swap to create the foreign currency debt synthetically**.

**Example**: US companies have sometimes found foreign investors reluctant to accept the risk of a corporate takeover, so it is less expensive to issue synthetic foreign currency debt by combining a swap position with actual US dollar debt issued to US investors. Example are found in Disney’s synthetic yen financing and in Fannie Mae’s multicurrency financing, described in the boxes.

**Note**: In a market where there are no inefficiencies and all securities are properly priced, the cost of actual US dollar debt and synthetic US dollar debt through a swap should be the same.

**Example**: your firm’s US dollar cost of US dollar debt is 6.50%. If you issue yen debt and then take a long yen position in a currency swap, you are effectively swapping the yen debt into synthetic US dollar debt. **After you take the swap position, the debt effectively becomes US dollar debt, even if it is synthetic**. **The interest rates built**
into the swap will force the cost of synthetic US dollar debt to be the same as actual US dollar debt. In reality, there may be some market inefficiency that allows you to end up with a lower “all-in” rate, but you should view this as a gain from shrewd financial engineering, rather than as a lower required rate of return by lenders.

**Settlement of Swap Components with Difference Checks**

Although **the idea behind a swap is to exchange cash flow streams**, the settlement of swap cash flows is often accomplished using difference checks.

**Example**: five-year fixed-for-fixed currency swap of 6% US dollars for 4% Swiss francs for a Notional Principal of $1000.

At the time-0 spot FX rate of 1.60 Sf/$,

- The currency swap is effectively an exchange of $60 for Sf 64 each year for five years
- Then a final exchange of $1000 for Sf 1600 at year 5.

The long Swiss franc position has contracted to pay the $60 and receive Sf 64 each year for five years, and to pay $1000 and receive Sf 1600 at year 5.

In fact, each swap cash flow is typically settled by a difference check based the actual spot FX rate. If the spot FX rate has moved to 2 Sf/$ as of the time of the first coupon component, the Sf 64 coupon interest at time 1 would be worth Sf 64/(2 Sf/$) = $32. The long Swiss franc position is scheduled to pay $60, but the US dollar value of the Sf 64 to be received is now $32. The scheduled payment may thus be settled simply by a difference payment of $60 – 32 = $28, from the counterparty that is long Swiss francs to the counterparty that is short Swiss francs.

**A difference check in a currency swap works in a fashion similar to a forward FX contract.**

The forward FX rate is replaced by an **FX conversion rate** for a component swap payment. **There are generally two different FX conversion rates in a fixed-for-fixed swap:**

1. one for **interest components**,
2. one for the **principal component**.

**FX Conversion Rate: Interest Components**, $C_i^{Sf/\$}$, is the spot FX rate times the ratio of the coupon interest rates equation (10.1a):

\[
FX \text{ Conversion Rate for Interest Component} \quad C_i^{Sf/\$} = X_0^{Sf/\$} \left( r_i^{Sf} / r_0^{\$} \right) \quad (10.1a)
\]

**FX Conversion Rate: Principal Component**, $C_p^{Sf/\$}$, is simply the spot FX rate at the time the swap originated, equation (10.1b):

\[
FX \text{ Conversion Rate for Principal Component} \quad C_p^{Sf/\$} = X_0^{Sf/\$} \quad (10.1b)
\]
Example: In the US dollar-Swiss franc swap, the FX conversion rates are:

**Interest Component:**  \[ C_{i \text{Sf$/S}} = (1.60 \text{ Sf$/})(0.04/0.06) = 1.0667 \text{ Sf$/}. \]

**Principal Component:**  \[ C_{P \text{Sf$/S}} = 1.60 \text{ Sf$/}. \]

[The numerator coupon interest rate should be consistent with the numerator currency.]

The **Difference Check Settlement** reveals that a swap exchange is essentially a forward FX contract.

The **FX Conversion Rate** is the Forward FX Rate;

the **Size (Z)** is the Foreign Currency Component, and

the **Amount (A)** is the Pricing Currency Component.

The **Difference Check At Time N** from the point of view of the long foreign currency position may be computed using equation (10.2), which is the same as equation (3.1) for forward FX contracts, but using the FX conversion rate (in direct terms from the US dollar point of view) in the place of the forward FX rate:

\[
D_{\text{Sf}} = Z_{\text{Sf}}(X_{N \text{Sf$/S}} - C_{\text{Sf$/S}}) \tag{10.2}
\]

**Example:** US-Swiss fixed-for-fixed currency swap,

The contract size (\(Z_{\text{Sf}}\)) for the interest component at time 1 is Sf64, and

The FX conversion rate is 1.0667 Sf/$.

The spot FX rate at the time of settling the interest component is 2 Sf/$,

Equation (10.2) indicates the long Swiss franc position should receive a **Difference Check Settlement:**

\[
Sf 64[1/(2 \text{ Sf$/}) – 1/(1.0667 \text{ Sf$/})] = Sf 64(0.50 \text{ $/Sf – 0.9375 $/Sf}) = –$28
\]

**Negative sign:** the long Sf position must pay $28 to the counterparty that is long US $.

Remaining interest components are settled using the spot FX rates at the time of payment.

**The Difference-Check Settlement for the PRINCIPAL COMPONENT uses the spot FX rate at the time the swap originated as the FX Conversion Rate in equation (10.2).**

The **Contract Size (Z_{\text{Sf}})** for the principal is the swap’s Notional Principal in foreign currency.

**Example:** the spot FX rate at the time of principal component settlement is 2.50 Sf/$.

The counterparty short Swiss francs owes Sf1600 equivalent to Sf1600/(2.50 Sf/$) = $640.

The long position in Swiss francs owes a **Notional Principal** of $1000, (?)

The counterparty long in Swiss francs must send a difference check for $360 to settle the **Principal Component** at maturity.

(Eq. 10.2) says the long position in Swiss francs is entitled to:

\[
D_{\text{Sf}} = Z_{\text{Sf}}(X_{N \text{Sf$/S}} - C_{\text{Sf$/S}}) = \text{Sf 1600} \left[ \frac{1}{2.50 \text{ Sf$/}} - \frac{1}{1.60 \text{ Sf$/}} \right] = –$360
\]
Since the sign is negative, the long position in Swiss francs settles with a difference check for $360 sent to the short position in Swiss francs.

**Because you can view each component of a fixed-for-fixed currency swap as a forward FX contract, you can interpret the swap as a portfolio of forward FX contracts. The forward FX rates in the portfolio of forwards that constitute the swap, however, are the swap contract’s FX conversion rates, not the market’s forward FX rates for the individual horizons.**

1. Consider the same swap as in #1, Three-year fixed-for-fixed currency swap of 5% US dollars for 2% Japanese yen. The spot FX rate was 112.50 ¥/$ when the swap originated. The notional principal is $1 million.

   **A)** What is the FX conversion rate for the interest component settlements?

   **B)** Find the difference check settlement if the yen depreciates to 120 ¥/$ at time 1 (the time of the first payment), and state which counterparty gets the check.

   had these payments

   Interest rate of market yield to find yearly coupon payment: (¥112,500,000)(0.02) = ¥2.25 mil

   Interest rate of market yield to find yearly coupon payment: ($1,000,000)(0.05) = $50,000

   find the embedded forward rate:

   \[
   \frac{¥2.25 \text{ mil}}{¥50,000} = 45 \frac{¥}{S}
   \]

   **Short Yen (diff check)**

   **Difference Check At Time N:**

   \[
   D_{\text{Sf}}^S = Z^S(X_N^{S/Sf} - C^{S/Sf}) = ¥2.25 \text{ mil} \left[ \frac{1}{120 \frac{¥}{S}} - \frac{1}{45 \frac{¥}{S}} \right] = -¥31,250
   \]

   **Long Yen (direct solution)**

   \[
   \frac{¥2.25 \text{ mil}}{120 \frac{¥}{S}} = ¥18,750
   \]

   The FX conversion rate is 45 ¥/$. The short position on yen gets ¥31,250.

3. What would be the settlement of principal at maturity of the swap in the previous problem if the spot FX rate at that time is 120 ¥/$?

   120 ¥/$: spot rate at settlement  
   112 ¥/$: spot rate when begun

   find the embedded forward rate: \[
   \frac{¥2.25 \text{ mil}}{¥50,000} = 45 \frac{¥}{S}
   \] from \[
   C_{\text{P}}^{S/S} = X_0^{S/S}
   \]

   **Difference Check At Time N:**

   \[
   D_{\text{v}}^S = Z^S(X_N^{S/Y} - C_{\text{P}}^{S/Y}) = ¥2.25 \text{ mil} \left[ \frac{1}{120 \frac{¥}{S}} - \frac{1}{45 \frac{¥}{S}} \right] = -¥31,250
   \]

   \[
   ¥112.5 \text{ mil} \left[ \frac{1}{120 \frac{¥}{S}} - \frac{1}{112.5 \frac{¥}{S}} \right] = -¥62,500
   \]
**Principle Component:**  
\[ D_y^s = Z_y (X_N^{s/y} - C_P^{s/y}) = \]

The net payment to the short position on yen is $62,500.
Consider a six-year fixed-for-fixed currency swap of 5% US dollars for 7% Japanese yen on notional principal of $1 million. Spot FX rate when the swap originates is 140 ¥/$.

A) Find the difference check settlement on the interest component if the spot FX value of the yen depreciates to 160 ¥/$ at time 1.

The Notional Principal in yen is $1 million (140 ¥/$) = ¥140 million.
The Notional US Dollar Interest is 0.05($1 million) = $50,000, while
The Notional Yen Interest is 0.07(¥140 million) = ¥9.80 million.
The FX Conversion Rate for the interest components is thus ¥9.80 million/$50,000 = 196 ¥/$.

At 160 ¥/$, the Interest Component settles (from the viewpoint of the long position on yen) at:

Interest Component: ¥9.8 million[1/(160 ¥/$) - 1/(196 ¥/$)]
= $11,250.

B) Find the difference check settlement on the principal component at time 6 if the spot FX rate then is 160 ¥/$.

At time 6:
The settlement of the principal component (from the viewpoint of the long position on yen) is:

¥140 million[1/(160 ¥/$) - 1/(140 ¥/$)] = $125,000.
to settle the interest component at time 1:
The long position on yen would receive $11,250 from the short position in yen
to settle the principal at time 6:
The short position in yen would receive $125,000 from the long position in yen
(equivalently $125,000 (160 ¥/$) = ¥20 million)

4. Consider a six-year, fixed-for-fixed currency swap of 5% US dollars for 8% British pounds at
The current spot FX rate of 1.60 $/£, on
Notional Principal of $1 million.

What are the two final difference checks, Interest and Principal, if
The spot FX rate is 1.80 $/£ at that time. State the direction of each check.

The Notional Principal in pound is $1 million (1/1.6 $/£) = £625,000.
The Notional US Dollar Interest is 0.05($1 million) = $50,000, while
The Notional Pound Interest is 0.08(£625,000) = £50,000.
The FX Conversion Rate for the interest components is thus
£50,000/$50,000 = 1 £/$.

At 1 £/$, the Interest Component settles (from the viewpoint of the long position on pound) at:
The Interest Component: £50,000 \[1.80 \, \text{$/£} - 1/(1 \, \text{£/$})\] = $40,000.
(six-year, fixed-for-fixed)
The settlement of the principal component (from the viewpoint of the long position on pound) is: ¥140 million\[1/(160 \, \text{¥/$}) - 1/(140 \, \text{¥/$})\] = $125,000.

Difference Check At Time N: \(D_\text{£} = Z_\text{£} \, (X_N^{$/£} - C_P^{$/£}) = \text{£625,000} \, [1.80 \, \text{$/£} - 1.60 \, \text{$/£}] = $125,000\)

The long position on pounds receives $40,000 to settle the interest component. The principal on the swap at maturity is settled with a difference check of $125,000 to the long position on pounds.

**Mark-To-Market Valuation of Currency Swaps**

Like other financial instruments, currency swap positions have a mark-to-market value that fluctuates. The MTM value of an AT-MARKET SWAP at time 0 is zero. After time 0 a swap position’s MTM value fluctuates continuously with changes in the spot FX rate and with changes in interest rates in the two currencies.

**To find the MTM value of a currency swap position:** take the present value of the remaining future inflows and subtract the present value of the remaining future outflows, using the spot FX rate to compare the present values in a common currency.

Consider the long Swiss franc position of a fixed-for-fixed currency swap as a combination of owning a coupon bond in Swiss francs and issuing a coupon bond in US dollars. Thus the MTM value of the long Swiss franc position, \(M_{\text{Sf}}\), may be viewed as the swap’s one-sided value of the notional Swiss franc bond, \(W_{\text{Sf}}\), minus the one-sided value of the notional US dollar bond, \(W_{\$}\), as shown in equation (10.3).

\[
\text{MTM Value of Currency Swap Position} \quad M_{\text{Sf}} = W_{\text{Sf}} - W_{\$} \quad (10.3)
\]

**Example:** what would be the MTM value just after the second interest settlement of the long Swiss franc position in a five-year, $1000, 6% US dollar for 4% Swiss franc currency swap that originated as an at-market swap when the spot FX rate was 1.60 Sf/$?

To answer you would need to know (at the time immediately after the second interest component)

1. the spot FX rate, and
2. the market yields of both currencies for a horizon equal to the remaining life of the swap.

In this case, the swap has three years left, as we are saying that the second interest component has just been settled.

To focus only on the influence of the FX rate change, let us assume that after the time-2
coupon interest component settlement, the market yields on three-year coupon bonds are 6% in US dollars and 4% in Swiss francs, the same as the original coupon rates of the swap.

With three years left, the present value of the US dollar payments of $60 for three more years, plus the principal payment of $1000, given the 6% market yield, is the one-sided value of the US dollar bond: \( W_\$ = 1000 \left( = \frac{60}{1.06} + \frac{60}{1.06^2} + \frac{1060}{1.06^3} \right) \)

The PV of the payments of Sf 64 for 3 years plus the Sf 1600 principal payment at 4% yield is:
\[
\text{Sf 64}/1.04 + \text{Sf 64}/1.04^2 + \text{Sf(64 + 1600)}/1.04^3 = \text{Sf 1600}.
\]

If the spot FX rate is \(1.60 \text{ Sf} / \text{S} \) at time 2, the one-sided value in US dollars of the Swiss franc bond, \(W_{\text{Sf}}\), is $1000 and the long Swiss franc swap position’s MTM value is: 
\[M_{\text{Sf}} = W_{\text{Sf}} - W_\$ = 1000 - 1000 = 0\] (the present values of the two sides of the underlying cash flows, $1000 and Sf 1600, balance each other at that spot FX rate).

If the spot FX rate at time 2 is \(1.25 \text{ Sf} / \text{S} \) the Sf 1600 present value of the future Swiss franc cash flows would be equivalent to a US dollar value of Sf1600/(1.25 Sf$/S) = $1280. Thus the long Swiss franc position owns a future cash flow stream of Swiss francs (equivalent to a Swiss franc bond) that has a one-sided value of \(W_{\text{Sf}} = 1280\), but owes a cash flow stream in US dollars (equivalent to a US dollar bond) with a present value of \(W_\$ = 1000\).

The MTM value of the swap: $1280 – 1000 = $280 (viewpoint of the long Swiss franc position).

The MTM value for the short Swiss franc position is the negative of the MTM value for the long Swiss franc position, in this case the MTM value of the short Swiss franc position is –$280.

In the second case, the Swiss franc has appreciated from the time the swap was originated, from 1.60 Sf$/S to 1.25 Sf$/S, and the long position on Swiss francs has thus gained overall economic value in the amount of $280.

If the long position on Swiss francs wants to liquidate in the open market, essentially finding a third party to assume the long position, the third party would have to pay $280 in order to take over the long swap position.

In the US dollar-Swiss franc example,

1. **What is the MTM value of the long Swiss franc swap position after the second coupon interest component payment, if the spot FX rate is 2 Sf$/S, but all else remains the same?**

After the second interest component settlement, the present value of the remaining Swiss franc cash flows, is Sf1600, which is equivalent to Sf1600/(2 Sf$/S) = $800. Thus, the MTM value of the swap from the viewpoint of the long position on Swiss francs is $800 – 1000 = –$200. The counterparty that is currently long on Swiss francs would have to pay $200 to
be able to turn the swap position over to a third party, given the assumed spot FX rate of 2 Sf/$ and the time-2 US dollar and Swiss franc interest rates.

2. What is the MTM value of the short Swiss franc position?

In this case, the Swiss franc has depreciated since the swap originated, and the swap’s long position on Swiss francs declines by $200 in value because of the depreciation. The short Swiss franc position’s MTM value is $200.

5. What is the MTM value, after three payments, of the long yen position in a six-year $1 million 5% fixed US dollar versus 2% fixed yen currency swap that originated as an at-market swap when the spot FX rate was 110 ¥/$?

Assume a spot FX rate of 120 ¥/$ at time 3, and that the market yields on three-year coupon bonds are 2% in yen and 5% in US dollars at time 3.

Setup as present value to a party who is long yen. What is it you have left to receive?

The notional values are the two interest components:

The Notional US Dollar Interest is \( 0.05 \times (\$1 \text{ million}) = \$50,000 \),

while

The Notional Yen Interest is \( 0.02 \times (110 \text{¥/$}) \times (\$1 \text{ million}) = ¥2,200,000 \).

Will receive ¥2.2mil @ t1, ¥2.2mil @ t2, ¥2.2mil @ t3, … ¥2.2mil @ tn + prin @ tn

need the yield in order to discount. Let’s find the present value of yen payments:

\[
\frac{¥2.2mil}{(1 + .02)} + \frac{¥2.2mil}{(1 + .02)^2} + \ldots + \frac{¥2.2mil}{(1 + .02)^3} + \text{prin} = ¥110\text{mil}
\]

Because in this case market yield = coupon yield the ¥110 million is already in present value terms of what I will receive.

Need US $ PV at 5%:

\[
\frac{¥50,000}{(1 + .05)^1} + \frac{¥50,000}{(1 + .05)^2} + \frac{¥50,000}{(1 + .05)^3} + \frac{¥1,000,000}{(1 + .05)^3} = ¥1,000,000
\]

= ¥110$mil$ = ¥916,666

Need US $ PV at 5%:

\[
\frac{¥110$mil$}{120¥} = ¥916,666
\]

Use formula 10.3 for MTM value:

\[
M_y^S = W_y^S - W_s^S = ¥916,666 - ¥1,000,000 = -¥83,333
\]

now we want % change in FX of yen:

This is depreciation of yen. In this case it is a 1-to-1 correspondence, down $8.3 when yen is down 8.3%.
Off Market: pay different up front, you arrange payments different from those offered in the market.

The swap has an MTM value in US dollars of −$83,333.
Off-Market Swaps

**Off-Market Swap**: two parties agree at time 0 to exchange cash flow streams that do not have the same present value, given the current spot FX rate. In this case, the recipient of the cash flow stream with the higher present value must make some time-0 balancing payment to the party receiving the cash flow stream with the lower present value. The time-0 balancing payment would equalize the present value of the exchange.

An **off-market swap** requires a time-0 payment to balance the present values.

---

**Example**: the time-0 spot FX rate is 1.60 $/£.
You want to make the future payments on a five-year, 5.50% coupon, US dollar par bond,
You want to receive payments on a five-year, 10% coupon, sterling bond.
The market yield to maturity on five-year, 10% coupon sterling bonds is 9%, not 10%.

1. **Would you**, with the long position on sterling, **pay or receive a time-0 payment?**
   For a **notional principal of $1000**, the **sterling notional principal is £625** (because the time-0 spot FX rate is assumed to be 1.60 $/£).
   The **10% sterling coupon rate** means that you’ll receive \(0.10(£625) = £62.50\) per year **for five years**, in addition to the **principal repayment of £625 at year 5**.
   The present value of the underlying sterling bond payments is:
   \[£62.50/1.09+£62.50/1.09^2+£62.50/1.09^3+£62.50/1.09^4+£62.50/1.09^5+£625/1.09^5 = £649.30\]
   which is equivalent to a US dollar value of **£649.30(1.60 $/£) = $1039**.

2. **How much is the payment?**
The **PV** of the US dollar cash flows you’d be paying is $1000.
The present value of the sterling cash flows you’d be receiving is equivalent to $1039.
Thus, you should make a time-0 payment of $1039 – 1000 = $39.

**Now in the same example;**
**What is the off-market time-0 swap payment necessary to be paid to you (or by you), if you have the long sterling position, and if you want a coupon payment of 8%, all else the same?**
**Answer:**
8% **sterling coupon rate** means that you’ll receive **0.08(£625) = £50** per year **for five years**,
in addition to the **principal repayment of £625 at year 5**.
The **PV** of the underlying sterling bond payments is:
\[£50/1.09 + £50/1.09^2 + £50/1.09^3 + £50/1.09^4 + £50/1.09^5 + £625/1.09^5 = £601\]
which is equivalent to a **US dollar value of £601(1.60 $/£) = $962**.
The **PV** of the US dollar cash flows you’d be paying is $1000,
The **PV of the sterling cash flows you’d be receiving is equivalent to $962**.
Since you are willing to accept future receipts at a below-market coupon rate, you should receive a time-0 payment of $1000 – 962 = $38.
Currency Swaps and FX Equity Exposure

Currency swaps will have an impact on a firm’s FX equity exposure just like foreign currency debt does. **When a firm combines a currency swap position with debt to engineer synthetic debt in a different currency, we regard the synthetic debt as the real debt for purposes of applying equation (9.2) to compute FX equity exposure.**

\[
\xi_{SE} = \frac{\xi_{VE} - \frac{ND_{E}S}{V^S}}{1 - \frac{ND^S}{V^S}} \tag{9.2}
\]

**Naked Currency Swap Positions:** a firm uses a currency swap on its own and not in order to create synthetic debt. When a firm has naked currency swap positions we can adapt equation (9.2) into equation (10.4).

\[
\xi_{SE} = \frac{\xi_{VE} - \frac{NL_{E}S}{V^S}}{1 - \frac{ND^S}{V^S}} \tag{10.4}
\]

Equation (10.4) is identical to equation (9.2) except that in the numerator the term \(NL_{E}S\) replaces \(ND_{E}S\). **NL_{E}S** represents the net intrinsic value of all the firm’s euro-denominated liabilities, including both actual euro debt and the one-sided values of off-balance sheet derivatives positions in the exposure currency. **To determine NL_{E}S when naked currency swap positions are used, the one-sided values of short euro currency swap positions would be added to any actual euro debt, while the one-sided values of long euro swap positions would be subtracted.**

**Example:** a firm has actual euro debt with a value of $20 million and a **Short euro position** in a currency swap with a one-sided value of $15 million, then \(NL_{E}S = $35\) million.
If a firm has actual euro debt with a value of $20 million and a **Long euro position** in a currency swap with a one-sided value of $15 million, then \(NL_{E}S = $5\) million.

The numerator of (eq. 10.4), \(NL_{E}S/V^S\), must be the ratio of net euro liabilities (actual debt and swap positions) to the firm’s intrinsic enterprise value.

The denominator of (eq. 10.4), \(ND^S/V^S\), should be the ratio of only the actual net debt (on the balance sheet) to the firm’s intrinsic enterprise value. It measures the firm’s financial
leverage.

Example: firm XYZ Co. has an intrinsic enterprise value of $2000, $200 of actual euro-denominated debt, $200 of actual non-euro-denominated debt, and thus an intrinsic equity value of $1600.

It has a naked short currency swap position on euros with a notional principal of $1000. The swap currently has no MTM value. (sum is $0, see above)

XYZ has an FX enterprise exposure to the euro of 2.

What is XYZ’s FX equity exposure using (eq. 10.4)?

The currency swap has no MTM value at time 0

→ the value in US dollars of the euro liability side of the swap is $1000.

Thus, $NL^{e}$/$V^{S}$ = ($200 + $1000)/$2000 = $1200/$2000 = 0.60, and $ξ_{sc}^{S}$ = (2 – 0.60)/(1 – 0.20) = 1.75.

by virtue of the FX enterprise exposure of 2, if the FX price of the euro falls by 5% the firm’s intrinsic enterprise value falls to $1800.

The value of the actual euro-denominated debt (in US dollars) falls to $190, while

The intrinsic value of the actual US dollar debt stays at $200.

Before considering the currency swap position, the new intrinsic equity value would be $1410.

When the euro drops 5% in FX price the euro side of the swap position will have a new value of $950.

The currency swap position is short euros, so there is an MTM gain of $1000 – 950 = $50. With this gain, the new intrinsic value of XYZ’s equity is $1410 + 50 = $1460.

The FX equity exposure computed directly is:

$$\left(\frac{1460}{1600} - 1\right) \div -0.05 = 1.75$$

which reconciles with the one found using equation (10.4).
To visualize what is going on in the example consider the two INTRINSIC BALANCE SHEETS before and after the FX rate change in Exhibit 10.2.

One-sided values of the currency swap are shown above the dashed line as off-balance sheet. The one-sided value of the long US dollar side is shown on the asset side, while the one-sided value of the short euro side is shown on the liability side.

We put the MTM value of the swap below the dashed line onto the intrinsic balance sheet, with net MTM gains shown as an asset and net MTM losses as a liability.

There is no MTM value on the time-0 balance sheet because the swap originated as an AT-MARKET SWAP.

---


**Time 0**

<table>
<thead>
<tr>
<th>NET DEBT &amp; EQUITY</th>
<th>ASSETS (Off)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$1000 (Long $, $W$)</td>
</tr>
<tr>
<td></td>
<td>$200 €-Net debt</td>
</tr>
<tr>
<td></td>
<td>$1600 Equity</td>
</tr>
<tr>
<td></td>
<td>$2000 V$</td>
</tr>
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</table>

**After Euro Drops by 5% (Time 1)**

<table>
<thead>
<tr>
<th>NET DEBT &amp; EQUITY</th>
<th>ASSETS (Off)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$1000 (Long $, $W$)</td>
</tr>
<tr>
<td></td>
<td>$190 €-Net debt</td>
</tr>
<tr>
<td></td>
<td>$1460 Equity</td>
</tr>
<tr>
<td></td>
<td>$1800 V$</td>
</tr>
<tr>
<td></td>
<td>$50 (Gain on Swap, Me$)</td>
</tr>
</tbody>
</table>
ABC Co. currently has:
$V^S = $5000,
$ND^S = $3000, and thus

Assume an FX enterprise exposure of 1.40 to the euro. $2000 of the firm’s net debt is euro-denominated, and the rest (of the net debt) is US dollar denominated. ABC has a naked long euro position in a currency swap with notional principal of $1000. The swap is currently an at-market swap.

What is ABC’s FX Equity Exposure using equation (10.4)? Assume the euro drops by 10% in FX price.

The net amount of euro liabilities is $NL^€ = $2000 – $1000 = $1000.
Thus $NL^€/V^S = $1000/$5000 = 0.20.
$ND^S/V^S = $3000/$5000 = 0.60.

Thus we have that $\xi^S_{e^S} = (1.40 – 0.20)/(1 – 0.60) = 3.$

If the FX price of the euro falls by 10%, the intrinsic enterprise value falls by 14% to $4300, by virtue of the FX enterprise exposure of 1.40.

The value of the actual euro net debt (in US dollars) falls to $1800, while the value of the actual US dollar net debt stays at $1000.

Before considering the currency swap position, the new intrinsic equity value would be $1500.

The euro side of the currency swap position will have a new value of $900, when the euro drops in FX price by 10%.

Since the currency swap position is long euros, there is an MTM loss on the currency swap position: $900 – 1000 = –$100.

Find ABC’s new Intrinsic Equity Value and show the FX equity exposure directly.

With this loss, the new intrinsic value of ABC’s equity is $1500 – 100 = $1400.

We see that the FX equity exposure computed directly is:
($1400/$2000 – 1)/–0.10 = 3.

This reconciles with the FX equity exposure found using equation (10.4).
MANAGING FX EXPOSURE WITH CURRENCY SWAPS

A firm may have a long FX enterprise exposure that is too great to hedge through actual foreign currency-denominated debt. It may then want to consider **supplementing actual foreign-currency debt with a naked short currency swap position**.

To see how a currency swap works to hedge FX exposure in this case, let us return to the XYZ Co. and assume now that XYZ puts on a $3800 (notional principal) short euro position in a currency swap. Before any FX change, the intrinsic enterprise value of XYZ is $2000, and the on-balance sheet capital structure consists of $200 of euro-denominated net debt, $200 of non-euro-denominated debt, and $1600 of equity. In addition, there is now the off-balance sheet short euro currency swap position. **If the FX price of the euro falls by 5%, the firm’s intrinsic enterprise value falls to $1800 by virtue of its FX enterprise exposure of 2.** The US dollar value of the actual euro net debt falls to $190, while the value of the US dollar net debt stays the same at $200. Before considering the currency swap position, the new intrinsic equity value would be $1410, as before.

The **CURRENCY SWAP** position is short euros, so the position will have a gain of 5% of the value (in US dollars) of the euro side of the swap, when the euro drops in FX price by 5%. The gain on the currency swap position is thus $0.05($3800) = $190. With this gain, the intrinsic value of XYZ's equity is $1410 + 190 = $1600 after the 5% drop in the FX price of the euro, exactly the intrinsic equity value prior to FX change. In this scenario, the $200 actual euro net debt level (ND€) and the $3800 short currency swap position combine to hedge the firm’s FX operating exposure (of 2) to the euro, so that the firm’s FX equity exposure is zero, all things considered.

The key is to make the total net amount of euro liabilities, NL€ (including both the actual euro debt and the off-balance sheet short euro position of the currency swap), equal to the FX enterprise exposure times the intrinsic enterprise value, \( \xi_{Ve}V^d \).

In this case, \( \xi_{Ve} = 2 \) and \( V^d = $2000 \); thus the total euro net liability level in US dollar terms, including the actual euro debt level and the short swap position’s notional principal, should be:

\[
NL\xi_{Ve}V^d = 2($2000) = $4000
\]

Since XYZ already had $200 in actual euro-denominated net debt, the hedge is completed by the $3800 short currency swap position on euros.
ABC Co. currently has:
V$ = $5000,
ND$ = $3000, and thus
S$ = $2000.

FX enterprise exposure to the euro = 1.60.
$2000 of the firm’s net debt is euro denominated, and
the remaining net debt is US dollar denominated.

**Determine the NOTIONAL PRINCIPAL of a currency swap position that will eliminate
ABC’s FX equity exposure.**

The total amount of euro net liabilities should be
\[ NL_{\varepsilon} = \xi_{\varepsilon\varepsilon}(V_{\varepsilon}) = 1.60(\$5000) = \$8000. \]

Since the firm already has $2000 in actual euro debt, the (at-market) currency swap
should have a notional principal of $6000. Due to the FX enterprise exposure of 1.60, the
intrinsic enterprise value will fall by 16%, from $5000 to $4200, when the euro drops by
10% in FX price.

**Assume the euro drops by 10% in FX price. Show how the intrinsic equity value
stays at $2000.**

The US dollar value of the actual euro net debt will drop by 10% to $1800, while the value
of the other actual net debt stays at $1000. The new intrinsic equity value for the firm
would be $4200 – 1800 – 1000 = $1400 before considering the currency swap gain or
loss. The swap has a gain of $600, since the swap position is short on $6000 worth of
euros, and the euro depreciates in FX price by 10%. Thus this $600 gain on the swap
position would bring the firm’s intrinsic equity value back to $2000.

A firm with a **Negative FX Operating Exposure** can manage this exposure with a **long
position on the foreign currency in a currency swap.**

The notional principal should be equal to the absolute value of \( \xi_{\varepsilon\varepsilon}(V_{\varepsilon}) \).

**Example:** Fly By Night’s enterprise value is £100 million and its
FX enterprise exposure to the US dollar is –1.25.

Then Fly By Night could hedge its FX exposure, eliminating the FX equity exposure, via a
**long currency swap position on the US dollar with a notional principal of £125 million.**
DEF Co. currently has:
\[ V^\$ = $5 \text{ million}, \]
\[ ND^\$ = $3 \text{ million, and thus} \]
\[ S^\$ = $2 \text{ million}. \]
FX enterprise exposure = –1.60 to the euro.
All of the firm’s actual net debt is US dollar denominated.
Determine the currency swap position that will eliminate DEF’s FX equity exposure.

The total amount of euro liabilities should be:
\[ NL_e^\$ = \xi V_e^\$ (V^\$) = -1.60 ($5 \text{ million}) = -$8 \text{ million}. \]

To have a “negative liability” of $8 million in euros, the firm should take a
naked long currency swap position on euros with a notional principal of $8 million.
The intrinsic value balance sheets for DEF illustrate how the FX equity exposure is zero.

### DEF Co. Intrinsic Value Balance Sheets
with Off Balance Sheet Long Euro Swap Position

**Time 0**

<table>
<thead>
<tr>
<th>ASSETS (Off)</th>
<th>NET DEBT &amp; EQUITY</th>
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<tbody>
<tr>
<td>$8 \text{ million (Long } e, W_e^$) ]</td>
<td>$8 \text{ million (Short } $, W_s^$)</td>
</tr>
<tr>
<td>[ $5 \text{ million } V^$ ]</td>
<td>[ $3 \text{ million Equity} ]</td>
</tr>
<tr>
<td>$5 \text{ million}</td>
<td>$5 \text{ million}</td>
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</table>

**After Euro Drops by 10% (Time 1)**

<table>
<thead>
<tr>
<th>ASSETS (Off)</th>
<th>NET DEBT &amp; EQUITY</th>
</tr>
</thead>
<tbody>
<tr>
<td>$7.20 \text{ million (Long } e, W_e^$) ]</td>
<td>$8 \text{ million (Short } $, W_s^$)</td>
</tr>
<tr>
<td>[ $0.80 \text{ million (MTM Loss, } - M_e^$) ]</td>
<td>[ $2 \text{ million -$Net Debt} ]</td>
</tr>
<tr>
<td>[ $5.80 \text{ million } V^$ ]</td>
<td>[ $3 \text{ million Equity} ]</td>
</tr>
<tr>
<td>$5.80 \text{ million}</td>
<td>$5.80 \text{ million}</td>
</tr>
</tbody>
</table>
As a long-term multiperiod instrument, the currency swap is a better hedging device for FX exposure than a single, short-horizon derivative, such as a forward FX contract. One reason is that the entire settlement on the forward FX contract will have to be in cash, which might not be available.

**Example:** XYZ Co. (above) tries to hedge its FX enterprise exposure with a 1-year forward FX contract in the amount of $3800 instead of the currency swap. If the firm’s intrinsic enterprise value falls due to a drop in the FX price of the euro, the company is ok because it has an offsetting gain in the forward FX contract. **But if the intrinsic enterprise value rises due to an appreciation of the FX price of the euro, the company has an offsetting loss in the forward FX contract.**

While the gain on the firm’s intrinsic enterprise value offsets the loss on the forward FX contract, **the gain in the intrinsic enterprise value is not a cash inflow**, but the company will, in principle, need cash at time 1 to settle its loss on the forward FX contract.

It is possible that the cash settlement may be avoided by “rolling” the forward FX contract, but **Two problems still exist:**

1. One is that cash deposits, or margins, are often required, and these margins will be raised as the unrealized losses in forward FX contracts mount. Such **margin increases will be a cash drain for the hedging firm**, even if the forward FX contracts do not have to be fully settled in cash.

2. The other problem is the appearance of the unrealized losses on the firm’s books, which may create its own problems. Currency swap positions accomplish FX hedging without affecting the firm’s balance sheet at the time the swap originates. As the FX price of the euro changes, though, the MTM gains/losses will affect the balance sheet, since they affect the intrinsic equity value in the manner described.

We have been using FX enterprise value exposure as a convenient way to summarize the FX exposure of the firm’s stream of future cash flows. **A currency swap position may be viewed as hedging the impact of FX changes on the firm’s intrinsic enterprise value or on its future operating cash flows.**

If one wants to **take the view that a currency swap is hedging operating cash flows**, then the **Principal Component** presents a bit of a problem. Market innovation has solved this problem through a derivative called an **amortizing swap**, in which the swap cash flows are based on a level annuity instead of a fixed coupon bond.
Unlevering Estimated FX Equity Exposure

Suppose a US firm is estimated to have an **FX equity exposure of 0.80 to the euro**. The firm has a **capital structure** that is 20% actual euro-denominated **net debt** and 25% actual US dollar net debt, and the rest equity (55%).

The ratio of **total net debt to enterprise value**, $ND^S/V^S$, is 0.45.

The firm has a **naked short euro currency swap position** with a one-sided value equal to the value of the actual euro-denominated net debt.

We can apply equation (10.4) “in reverse” to **UNLEVER** the estimated FX equity exposure and to estimate the firm’s FX enterprise exposure:

$$
\xi_{r\epsilon}^S = \xi_{Se}^S (1 - \frac{ND^S}{V^S}) + \frac{NL_{\epsilon}^S}{V^S} = 0.8(1 - 0.45) + 0.40 = 0.84
$$

Double-check using equation (10.4) directly:

$$
\xi_{Se}^S = (0.84 - 0.40)/(1 - 0.45) = 0.80. \text{ (??)}
$$

ABC **Co. currently has:**
- $V^S = $5000,
- $ND^S = $2000, and thus
- $S^S = $3000.

**Estimated FX equity exposure of 0.30 to the euro.**

$1500 of the firm’s actual net debt is euro-denominated and the rest is US dollar net debt.

**What is the Estimated FX Enterprise Exposure using equation (10.4) if ABC also has a $2000 short euro currency swap position?**

$$
0.30 = \frac{\xi_{r\epsilon}^S - 0.70}{(1 - 0.40)} \Rightarrow \xi_{r\epsilon}^S = 0.88
$$
6. Assume DZD Co. currently has
\[ V^S = $5 \text{ million}, \]
\[ ND^S = $3 \text{ million}, \] and thus
\[ S^S = $2 \text{ million}. \]

**FX Enterprise Exposure** to the euro of 1.60. \[ \frac{V^S}{V^S} = 1.60 \]

$2 million of the firm’s actual debt is euro debt and
The rest is US dollar debt.
DZD takes an at-market currency swap position that is
short on euros with a notional principal of $2 million.

Find DZD’s **FX equity exposure to the euro** directly using a 10% depreciation of the FX price of the euro, and reconcile the answer using equation (10.4).

DZD’s intrinsic enterprise value will fall by 16% to $4.20 million.
The US dollar value of the euro-denominated actual net debt will drop by 10% to $1.80 million, while the US dollar value of the other actual net debt stays at $1 million.
The new intrinsic equity value for the firm: $4.20 million $1.80 million $1 million = $1.40 million before considering the currency swap MTM gain or loss.
The swap has a gain of $200,000, since the swap position is short on $2 million worth of euros, and the euro depreciates in FX price by 10%.
Thus the new intrinsic equity value is $1.60 million.
The firm’s FX equity exposure to the euro is ($1.60 million/$2 million $1)/(0.10) = 2.
To use equation (10.4), the total amount of euro liabilities (actual plus off balance sheet) is $4 million, and the ratio of euro liabilities to the enterprise value of the firm is $4 million/$5 million = 0.80.
Thus the firm’s FX equity exposure to the euro is $(1.60 0.80)/(1 0.60) = 2.

**UNLEVERING EQUITY BETAS**

In the basic case where a US firm’s debt is denominated entirely in US dollars and the firm has no off-balance sheet financial risk management positions, there is a simple relationship between the **Firm’s Equity Beta And Its Enterprise Beta**: \[ \beta_S^S = \beta_V^S/(1 - ND^S/V^S), \] equation (7.3).

This section upgrades our earlier formula for estimating the overall firm’s enterprise beta from its equity beta, using a procedure for removing the systematic risk effects of risk management positions and net debt denominated in foreign currency.

Equation (10.5) shows this relationship with the enterprise beta on the left-hand side.

\[ \beta_S^S = \beta_S^S(1 - \frac{NL^S}{V^S}) + \beta_e^S \left( \frac{NL_e^S}{V^S} \right) + \beta_L^S \left( \frac{NL_L^S}{V^S} \right) + \beta_y^S \left( \frac{NL_y^S}{V^S} \right) \ldots (10.5) \]
Equation (10.5) first adjusts the estimated equity beta for the overall degree of financial leverage of the enterprise, $ND^S/V^S$, the ratio of the net debt value to the enterprise value of the firm. (Both are shown measured in US dollars.) Any or all of the net debt may be denominated in any currencies. $ND^S$ does not include the notional principal amounts of any off-balance sheet currency swap positions, although it will include net accumulated mark-to-market gains/losses on swaps that appear on the balance sheet.

The next components of equation (10.5) adjust for any systematic FX risk in foreign currency debt or currency swap positions. $NL^S$ represents the value (in US dollars, at the current spot FX rate) of all euro-denominated liabilities, including both actual euro-debt and the notional value of currency swap positions on euros. As before, $\beta^S_\varepsilon$ is the currency beta of the euro. The notation is the same for pounds and yen in the next terms of equation (10.5). Liabilities in other currencies are represented in equation (10.5) by the three elliptical periods (...) . If the currency beta of a currency is positive, the use of foreign currency debt and swap positions in lieu of domestic currency debt reduces the equity beta. We add back in the systematic portion of the liabilities’ FX effect to the equity beta to calculate the estimated enterprise beta in US dollars.

**Example:** a US firm has only euro-denominated debt, representing 40% of the firm’s enterprise value, and has no other debt or currency swap positions, and the currency beta of the euro ($\beta^S_\varepsilon$) is 0.10. If we observe that the firm has an equity beta of 1.25, then (eq. 10.5) tells us that the enterprise beta (in US dollars) is $1.25(1 – 0.40) + 0.10(0.40) = 0.79$.

Next, assume instead that the firm is levered with actual euro-denominated debt representing 20% of the firm’s enterprise value and has a short at-market currency swap position on euros with a notional value representing 10% of the firm’s enterprise value (in US dollars). Now $NL^S_\varepsilon$ is 0.30, while $ND^S/V^S$ is only 0.20. If the equity beta is 1.25 and the currency beta of the euro ($\beta^S_\varepsilon$) is again 0.10, the firm’s enterprise beta would be $1.25(1 – 0.20) + 0.10(0.30) = 1.03$.

**XYZ Co.’s estimated equity beta (in US dollars) is 1.35.**

**XYZ’s equity value is $1400.**

**XYZ has US dollar net debt with a value of $200 and euro-denominated net debt with a value in US dollars (at the current spot FX rate) of $400. XYZ also has a short currency swap position on pounds, with a notional one-sided value (not MTM) in US dollars (at the current spot FX rate) of $200.**

The currency beta of the euro is 0.10 and The currency beta of the pound is 0.15.

**What is XYZ Co.’s Estimated Enterprise Beta (in US dollars)?**


The value of the actual debt, $600, is 30% of XYZ’s enterprise value.
Thus $\frac{ND^S}{V^S} = 0.30$.
The euro-denominated net debt with a value of $400$ has a beta of $0.10$.
The sterling liabilities (notional swap value) are $200$, $10\%$ of the firm’s enterprise value,
The currency beta of the pound is $0.15$.
(Eq. 10.5) gives: $\beta_V = 1.35(1 - 0.30) + 0.10(0.20) + 0.15(0.10) = 0.98$.

It may be helpful to think in the other direction and show the impact of capital structure on equity beta, given a firm’s enterprise beta.

**Example:** US firm XYZ Co. has an enterprise beta, $\beta_V^S$, of $0.90$.
XYZ has no swap positions and only
US $\$ debt representing $40\%$ of the firm’s enterprise value,
XYZ’s equity beta would be $0.90/(1 - 0.40) = 1.50$.

Now assume XYZ has only actual euro-denominated debt, again representing $40\%$ of the firm’s enterprise value, and has no other debt or currency swap positions, and the currency beta of the euro ($\beta_e^S$) is $0.10$.

Rearrange (eq. 10.5) to find that XYZ’s equity beta would be:

$$(0.90 - 0.10(0.40))/(1 - 0.40) = 1.433.$$  

Finally, assume instead that XYZ is levered with actual euro-denominated debt representing $20\%$ of the firm’s enterprise value and has a short currency swap position on euros with an one-sided value representing $10\%$ of the firm’s enterprise value (in US dollars).

Now $NL_e^S$ is $0.30$, while $\frac{ND^S}{V^S}$ is only $0.20$.
If the currency beta of the euro ($\beta_e^S$) is $0.10$,
XYZ’s equity beta would be: $$(0.90 - 0.10(0.30))/(1 - 0.20) = 1.0875.$$  

7. Assume ABC Co. currently has $V^S = 10$ million, $ND^S = 4$ million, and thus $S^S = 6$ million. Assume an FX enterprise exposure to the euro of $1.25$. Assume that $3$ million of the firm’s actual net debt is euro-denominated and the rest is denominated in US dollars. Determine the notional principal of a currency swap position that will eliminate ABC’s FX equity exposure. Assume the FX price of the euro rises by $10\%$. Show how the intrinsic equity value stays at $6$ million.

The total amount of ABC’s euro liabilities should be $1.25(10$ million) = $12.5$ million.
Since the firm already has $3$ million in actual euro-denominated net debt, the swap should have a notional principal of $9.50$ million.
The intrinsic enterprise value of the firm will rise by $12.5\%$ to $11.25$ million.
The US dollar value of the actual euro-denominated net debt will rise by $10\%$ to $3.30$ million, while the value of the other debt stays at $1$ million.
The firm’s new intrinsic equity value would be $11.25 million – 3.30 million – 1 million = $6.95 million before considering the swap gain or loss. The swap has a loss of $950,000, since the swap position is short $9.50 million worth of euros, and the euro appreciates by 10%. Thus the intrinsic equity value remains at $6 million.

8. Assume ABC Co. currently has $V^e = $10 million, $ND^e = $4 million, and thus $S^e = $6 million. Let us say all $4 million of the firm’s net debt is denominated in US dollars. Assume also that ABC has a long position on euros in a currency swap with a value of $2 million. Assume that ABC’s FX equity exposure to the euro is estimated to be 0.50. Find ABC’s estimated FX enterprise exposure to the euro.

$$\xi^e = \xi^e (1 – ND^e/V^e) + NL^e/V^e = 0.50(1 – 0.40) – 0.20 = 0.10.$$ Note that the long swap position on euros results in a negative value for ND^e/V^e.

9. XYZ Co.’s estimated equity beta (in US dollars) is 1. XYZ’s equity value is $6000. In addition, XYZ has US dollar net debt with a value of $1500 and pound-denominated net debt with a value in US dollars (at the current spot FX rate) of $500. XYZ also has a long currency swap position on euros, with a notional one-sided value (not MTM) in US dollars (at the current spot FX rate) of $1000. Assume that the currency beta of the euro is 0.10 and the currency beta of the pound is 0.15. What is XYZ Co.’s estimated enterprise beta (in US dollars)?

Note that the enterprise value of XYZ ($8000) is equal to the value of all of the actual debt ($2000) and the value of the equity ($6000). The value of the actual debt, $2000, is 20% of XYZ’s enterprise value. Thus $ND^s/V^s = 0.25$. The ratio of pound-denominated net debt to enterprise value is $500/$8000 = 0.0625. The long currency swap position on euros is a negative net liability, –$1000, which is –12.5% of the firm’s enterprise value. Thus, using equation (10.5), $\beta^e = 1(1 – 0.25) + 0.15(0.0625) + 0.10(–0.125) = 0.75$. 